

Giant Rings in the CMB Sky

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We find a unique direction in the CMB sky around which giant rings have an anomalous mean temperature profile. This direction is in very close alignment with the afore measured anomalously large bulk flow direction. Using Monte Carlo simulations, we estimate the significance of the giant rings at the 3σ level and the alignment with the bulk flow at 2.5σ . We argue that a cosmic defect seeded by a pre-inflationary particle could explain the giant rings, the large bulk flow and their alignment.

I. INTRODUCTION

One of the key assumptions in modern cosmology is statistical isotropy. The detailed data from the Wilkinson Microwave Anisotropy Probe (WMAP) provides an opportunity to test this assumption. Indeed many authors have studied this issue directly and indirectly using various approaches and claimed the existence of a number of anomalies in the data (see e.g. [1–21] and [22, 23] for recent reviews).

In this paper we propose another approach to test statistical isotropy. We study how much giant rings in the Cosmic Microwave Background (CMB) sky deviate from random behavior and estimate the significance of the deviation. In section II we define the *rings score* as a function of the direction the giant rings surround and generate a rings score map for the masked Internal Linear Combination (ILC) map. We show that the ILC rings score map has a clear peak. We estimate the giant rings in the ILC map as a 3σ deviation from Λ CDM. Moreover, we find that the giant rings are aligned with another reported Λ CDM anomaly [24–26] in the form of a large cosmic bulk flow.

In section III we discuss a cosmological scenario that could explain the giant rings, the large bulk flow and their alignment. It is this cosmological scenario, which involves the effects of a pre-inflationary particle [27, 28], that actually motivated us to look for these giant rings in the first place. Section IV is devoted to discussion.

II. GIANT RINGS IN THE SKY

We begin with the following question: Are there unusual rings in the CMB sky? For reasons that will become clear shortly, we choose to focus on the largest possible rings, namely those that reside in a band of width β around $\theta = \pi/2$ with respect to some direction specified by a unit vector \hat{n} (see Fig. 1). The band is symmetric

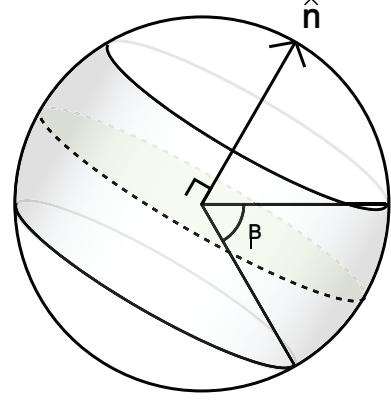


FIG. 1: The score is calculated for rings in a band of width β surrounding the equatorial defined by a direction \hat{n} .

and so the range of angles considered is

$$\frac{\pi - \beta}{2} < \theta < \frac{\pi + \beta}{2}. \quad (1)$$

We denote the average temperature of an infinitesimal ring by $T(\theta, \hat{n})$ and the mean of the map by T_0 and use the following *rings score* to detect unusual rings in the sky

$$R(\beta, \hat{n}) = \int_{\frac{\pi - \beta}{2}}^{\frac{\pi + \beta}{2}} d(\cos \theta) \tilde{T}^2(\theta, \hat{n}), \quad (2)$$

where $\tilde{T}(\theta, \hat{n}) = T(\theta, \hat{n}) - T_0$. This score is chosen since we are not looking to find any particular shape of $\tilde{T}(\theta, \hat{n})$. Rather, we are searching for the direction in which the rings deviate maximally from random gaussian behavior. For this we simply need to weigh correctly the contribution of each infinitesimal ring to our rings score. This is the reason for the $d(\cos \theta)$ in the score.

There are some issues one needs to deal with when working with actual CMB data. First, to have enough statistics in each infinitesimal ring, the rings cannot be taken to be arbitrarily small and the integral must be replaced by a discrete sum. In the results reported below for the 7-year ILC map (given in 1° resolution) we took $d\theta \rightarrow \Delta\theta = 3^\circ$, but have verified that the results are not sensitive to $\Delta\theta$. Secondly, for obvious reasons we would

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like the results to be insensitive to Galactic foregrounds. Hence we use the KQ75 mask which removes 29% of the WMAP7 sky and calculate the quantity

$$R_{\text{dis}}(\beta, \hat{n}) = \sum_{i=1}^{\beta/\Delta\theta} \tilde{T}^2(i, \hat{n}) M(i, \hat{n}), \quad (3)$$

where $M(i, \hat{n})$ is the number of pixels in the i 'th ring that survived the KQ75 mask cut and $\tilde{T}(i, \hat{n})$ is the difference between the average temperature in the i 'th ring around the direction \hat{n} and the mean of the masked map.

We would like to test the isotropy assumption of Λ CDM via the rings score. With this goal in mind β cannot be taken to be too small since this will increase the chance that the direction favored by $R_{\text{dis}}(\beta, \hat{n})$ has no significant importance and is merely a statistical fluke. However, due to the mask we are using we cannot take β to be too large either. The reason is that as we increase β the average size of an infinitesimal ring becomes smaller and so the ratio between the number of pixels we are masking and the pixels we are keeping when calculating $\tilde{T}(i, \hat{n})$ becomes larger for generic values of \hat{n} .

Note that even if β is small $R_{\text{dis}}(\beta, \hat{n})$ does not approximate $R(\beta, \hat{n})$ well when \hat{n} points roughly perpendicular to the galactic plane, because the corresponding rings lie mainly inside the mask. To be on the safe side, we ignore directions for which more than 80% of the rings have less than 30% unmasked pixels in each ring. Thus using the cut sky eliminates a small portion of the map around the north and south poles.

To verify that the defined rings score is not biased by the mask we used the HEALPix package to generate 100,000 random maps and masked them with the KQ75 map. We found that the peaks in the corresponding rings score maps are uniformly distributed on the sky and do not favor any particular area (see Fig. 2).

In Fig. 3 we plot the rings score calculated for the 7-year ILC map with the KQ75 mask for $\beta = \pi/6, \pi/3$ and

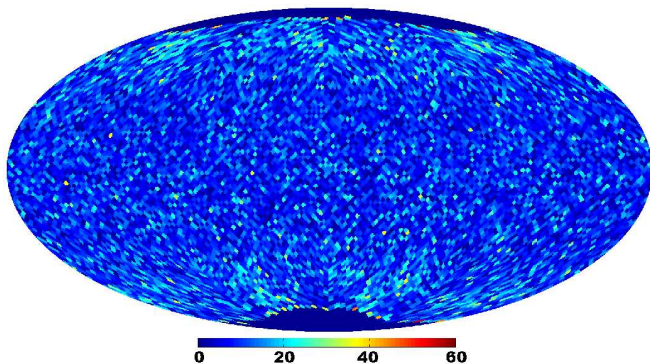


FIG. 2: A histogram showing the location of the maximum rings score (with $\beta = \pi/3$) for each of 100,000 randomly generated maps using the KQ75 mask. The maximum scores for 27% of the maps lie within the mask, which covers 29% of the sky.

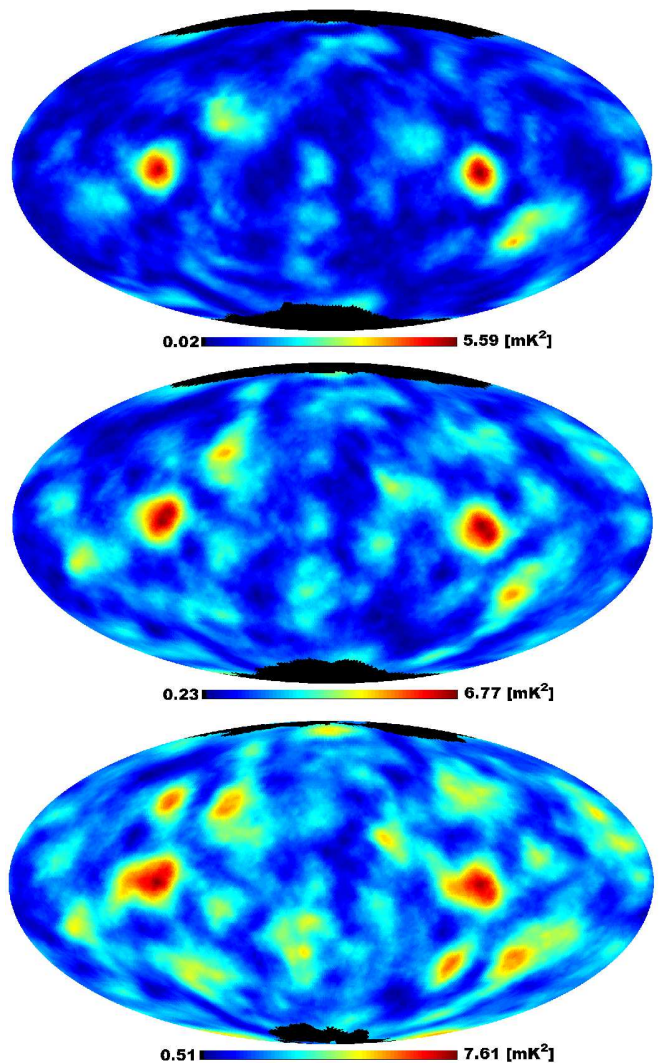


FIG. 3: *From top to bottom:* The rings score calculated for $\beta = \pi/6, \pi/3$ and $\pi/2$, respectively. The score is calculated on the 7-yr WMAP ILC map (degraded to $N_{\text{side}} = 64$) with the KQ75 mask applied. Ignored directions are marked in black.

$\pi/2$. The dark regions near the poles are ignored for the reason explained above. We see that at around Galactic coordinates $(l, b) = (276^\circ, -1^\circ)$ there is a distinct peak.¹ In fact, as illustrated in Fig. 4, even with the naked eye, the giant rings are visible.

There are some interesting aspects to this rings score map in general and the peak at $(276^\circ, -1^\circ)$ in particular. The location of the peak of $R_{\text{dis}}(\beta, \hat{n})$ is fairly insensitive to the value of β (as long as it is not too small or too

¹ Up to effects of the asymmetric masking the rings score is symmetric under a 180° inversion and so a symmetric partner peak appears as well. Taking an asymmetric θ range, however, the direction $(276^\circ, -1^\circ)$ is more prominent.

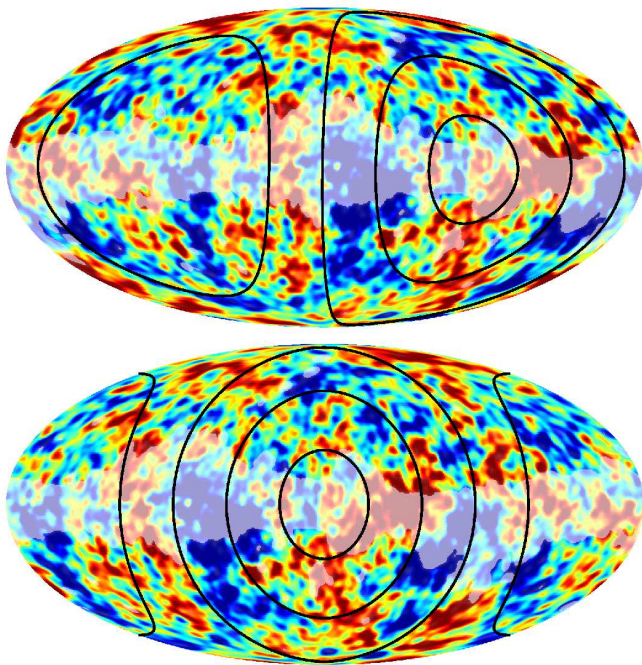


FIG. 4: *Top*: The 7 year ILC map, smoothed to 3° resolution. The KQ75 mask is faintly superimposed on the map and the rings are marked around the dominant direction. *Bottom*: The same map rotated so that the dominant direction is placed at the center of the map. Incidentally, two of the *Cold Fingers* discussed recently by the WMAP team [29] (the ones that include what [29] refer to as *Cold Spot I* and *Cold Spot II*) fall nicely inside the cold ring ($55^\circ < \theta < 85^\circ$), with their hot counterparts falling inside two surrounding hot rings ($25^\circ < \theta < 55^\circ$ and $85^\circ < \theta < 115^\circ$).

large). To be precise, when varying β from 30° to 165° , taking 10 evenly spaced values, the peak moves at most by 2° . Repeating this test on 10,000 randomly generated maps (masked with the KQ75 mask), only in 14 cases the peak was as stable. This implies a 3σ deviation from the statistically isotropic Λ CDM model.

As is evident from Fig. 3 most of the signal comes from the largest rings. To quantify this we calculate for each map ($\beta = \pi/6, \pi/3$ and $\pi/2$) the score

$$S(\beta) = \frac{R_{\max} - \bar{R}}{\sigma}, \quad (4)$$

where R_{\max} is the maximum of the rings score map and \bar{R} and σ are the mean and standard deviation of the rings score map and get $S_{\text{ILC}}(\beta) = 7.09, 5.85$ and 4.93 , respectively. We estimate the significance by calculating $S(\beta)$ on random maps and find that for $\beta = \pi/6, \pi/3$ and $\pi/2$ only $0.16\%, 0.57\%$ and 3.97% of the maps get a higher score, respectively. This implies that most of the signal comes from a narrow band around $\theta = \pi/2$, and that as we increase β we increase the noise without increasing the signal.

Several features breaking statistical isotropy have been found in WMAP data that turned out to be the result

of astrophysical or systematic effects and not of cosmological origin. Of course, we cannot completely rule out a similar explanation for the giant rings, but there is support for a cosmological explanation. First, the rings score maps for the V and W frequency bands are almost identical to that of the ILC (see Fig. 5). Secondly, the ecliptic pole is located at $(276^\circ, -30^\circ)$ and is $\sim 30^\circ$ away from both the main rings score peak and the secondary peak at $(248^\circ, -34^\circ)$.

Further support for the cosmological origin of the giant rings is the intriguing alignment between the direction of the rings and the direction of the large bulk flow reported in [25]. According to [25] the bulk flow on scales of about 100 Mpc/h has a magnitude of $|v| = 416 \pm 78 \text{ km/s}$ towards $(l, b) = (282^\circ \pm 11^\circ, 6^\circ \pm 6^\circ)$. The chance of such a large bulk flow to happen in Λ CDM on such large scales is about 0.5% [25]. As plotted in Fig. 6, for $\beta = \pi/3$ the rings score direction is $(276^\circ \pm 9^\circ, -1^\circ \pm 7^\circ)$ and the distance between it and the Bulk Flow direction is 9_{-9}^{+12} degrees. The probability of a 9° alignment between two random axes in the sky is 1.3% .

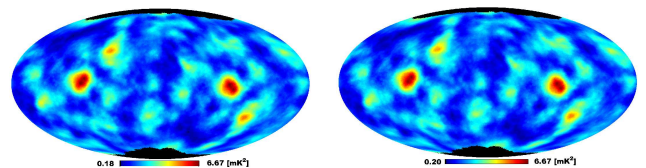


FIG. 5: $R_{\text{dis}}(\pi/3, \hat{n})$ for the foreground reduced 7-yr WMAP V (*left*) and W (*right*) frequency band maps (degraded to $N_{\text{side}} = 64$). The score maps are very similar to one another and to the ILC score map. In both maps, the peak is in the same location, $(276^\circ, -1^\circ)$ and its significance vs. random maps is $\sim 1\%$. The location remains the same also when tested in higher resolution of $N_{\text{side}} = 128$.

Overall, with current data we have two large scale quantities (one is large scale in terms of the CMB and the other in terms of Large Scale Structure) that are slightly anomalous that point roughly to the same direction. In Λ CDM there is no correlation between the bulk flow and the rings score and so a fair point of view is to attribute the alignment between the two to a statistical fluke (which is not so rare – a 1.3% effect) and to argue that both features are not anomalous enough to challenge Λ CDM.

In the next section we would like to offer a different point of view. We show that the scenario of [27, 28] naturally explains the anomalous bulk flow, the giant rings and their alignment. Taking the above results at face value, this scenario explains a one-in-a-million effect in Λ CDM.

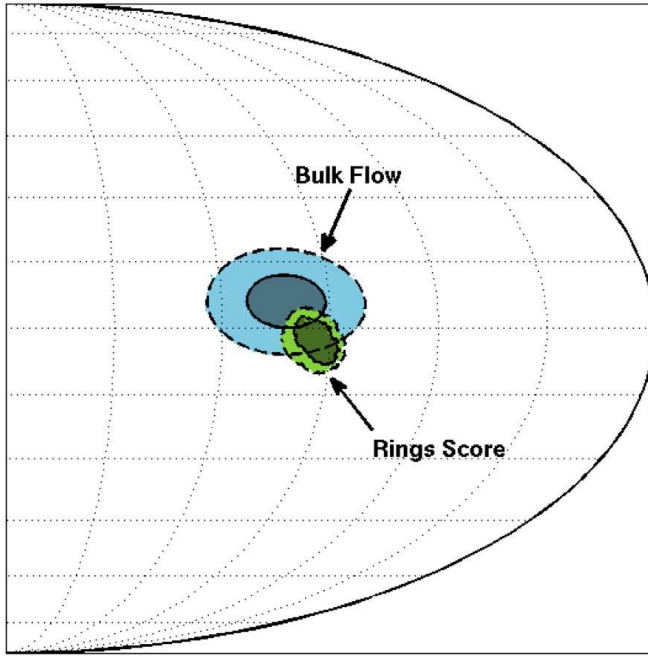


FIG. 6: The rings score direction ($276^\circ \pm 9^\circ$, $-1^\circ \pm 7^\circ$) and the Bulk Flow direction ($282^\circ \pm 11^\circ$, $6^\circ \pm 6^\circ$). The 1σ distance between the two is 9_{-9}^{+12} degrees. Areas for 1σ and 2σ are shown.

III. A POSSIBLE COSMOLOGICAL EXPLANATION

The fact that the bulk flow appears to be too large is known for quite some time now. What is fairly new is that the shear and octupole moments associated with the bulk motion appear to be consistent with Λ CDM [25] (see also [30]). This suggests that these higher moments are generated by the standard Λ CDM power spectrum, while the overall bulk flow is generated by a non- Λ CDM faraway source.

A model with exactly this feature (motivated by [31] and earlier works [32–34]) was suggested recently in [27, 28]. There, we studied some of the cosmological imprints of pre-inflationary particles (PIP). We found that each PIP provides the seed for a giant structure (a spherically symmetric Cosmic defect – SSCD) whose gravitational potential is determined by the PIP in the following way

$$\Phi_0(k) = \frac{\lambda H}{12\sqrt{\pi}\epsilon k^3} \Big|_{k=a(t)H}, \quad (5)$$

where ϵ is the slow roll parameter, $\lambda = dm/d\phi - m\sqrt{\epsilon/2}$ (with ϕ being the inflaton field) and, as usual, the effect is evaluated at horizon crossing.

For simplicity and concreteness it was assumed in [28] that $d\lambda/d\phi$ and $n_s - 1$ are negligible to find

$$\Phi(r, z=0) = \lambda C \log(r), \quad C = 1.09 \times 10^{-5}. \quad (6)$$

Relaxing these assumptions one typically finds

$$\Phi(r) \sim r^\alpha, \quad (7)$$

with $|\alpha| \ll 1$.

Both (6) and (7) vary very slowly over large distances and therefore are quite different than typical potentials generated by the Λ CDM power spectrum. As a result, such a SSCD has distinct cosmological imprints [28]. In particular, it can induce a large bulk flow from far away towards its direction while having negligible effect on higher moments of the bulk motion. Hence it fits neatly with the observations of [25].

The CMB signal of a single SSCD (seeded by a PIP) is affected by its magnitude (λ in the case of (6)) and its distance from the observer, denoted by z_0 . Setting the magnitude to produce the measured bulk flow for each z_0 , we remain with z_0 as a single free parameter. The CMB signal is made up of competing contributions from the Sachs-Wolfe (SW) and the late integrated SW (ISW) effects and as was shown in [28], it should be detectable in the CMB in the sense that it is larger than the noise. However, near $z_0 \sim 3$ the two nearly cancel out, so that a SSCD located there can account for the measured bulk flow while adding a low, but detectable, signal to the CMB sky that would not immediately stand out as an obvious violation of statistical isotropy.

So how does one search for the SSCD in the CMB data? In particular, we wish to find a way to tell apart the CMB signal of a SSCD from that of an unusually strong structure generated by the Λ CDM power spectrum.

An overdense Λ CDM structure will induce a cold *spot* in the CMB sky if located at the last scattering surface and a hot *spot* if located nearby. Because of the unique large distance behavior of (6) (or (7)), a SSCD will induce a more complex imprint that spreads all over the CMB sky (see [28]). The shape of this imprint is azimuthally symmetric and its profile depends on z_0 . Therefore its generic signature is a spot surrounded by rings. An interesting case happens where the contributions of the SW and ISW effects almost cancel out in the low multipoles (the cancellation happens at a different z_0 for each multipole). This can lead to the disappearance of the spot for certain z_0 values. Since the potential falls slowly with distance, the fact that the circumference is maximal at $\theta = \pi/2$ dominates and so a generic signature is the appearance of anomalous rings around $\theta \sim \pi/2$ from the location of the PIP. Focusing on these rings lowers the possibility that the score will confuse an atypical Λ CDM structure with a SSCD seeded by a PIP.

Moreover, if indeed a single SSCD is responsible for most of the bulk velocity observed in [25], then it should be located very near to the Galactic plane, where the small θ signal will be contaminated by Galactic foregrounds much more than the large θ profile. Even though part of the signal predicted by the SSCD lies in small angles, limiting the search and focusing only on large angles yields a signal that is cleaner both with respect to Galactic foreground and ordinary Λ CDM effects.

For these reasons we defined the azimuthally invariant rings score the way we did in the previous section. It is designed to detect a SSCD seeded by a PIP.

To verify that the score works as it should, we simulated random CMB maps that include the contribution from a SSCD located at some specific direction at a certain distance, and checked if the direction with the maximal rings score is indeed near the SSCD direction. This is done by calculating the SSCD temperature imprint from the SW and ISW effects (taken from [28]) and adding it to the randomly generated map (we calculate the effect in a multipole expansion and add the first non-vanishing 10 multipoles $a_{\ell 0}$ s to those of the random maps). The map is then rotated so that the SSCD is *hidden* at a desired direction. Next, we calculate the rings score on the map and check whether this direction indeed dominates the map. In Fig. 7 we plot the percentage of random maps with a hidden SSCD where the maximal rings score is less than 8° away from the hiding location. We see that, as expected, it resembles the S/N graph in [28] (Fig. 6(b)). A detailed study of the relation between the giant rings, other CMB anomalies and the PIP model will appear in a future work.

IV. CONCLUSIONS AND DISCUSSION

In this paper we reported on a novel unexpected feature of the CMB sky – giant anomalous rings. The significance of these giant rings by themselves, much like other “anomalous” features of the CMB, is far from being overwhelming as it is merely a 3σ effect. Moreover, much like the absence of large angular correlations [1, 2], our findings are weaker when considering the full ILC map with no mask – the peak in the rings score map remains aligned with the bulk flow direction, but the score is no longer significant vs. random maps. This could either mean that this is due to the contamination of the unmasked data or that the feature is weaker than suggested by the masked map. With data from Planck we should be able to determine which possibility is right. In fact, since Planck is expected to clean much of the Galactic foreground, we should be able to include the small θ

data as well and see if the signal increases.

Estimating the significance of features in the data is tricky in general and in the case of statistical isotropy in particular. Indeed many of the reported large scale anomalies in the CMB that imply violation of statistical isotropy were recently deemed as stemming from a-posteriori choices of estimators [29, 35–38] that amplified the significance of the results. Among the claims against these anomalies is that they surfaced from a search of oddities in the data with no independent experimental evidence or prior theoretical motivation.

The giant rings are different in this regard. First, the search for them was motivated by a theoretical scenario which by construction violates statistical isotropy. Secondly, they are aligned with another large scale *non* CMB “anomaly” – the bulk flow. This increases, in our opinion, the chance that our findings could eventually lead to a real challenge for statistical isotropy. For this to happen more data is needed.

Luckily there are two clear predictions of our scenario that could be tested already with Planck data. First, the weak gravitational lensing of the CMB by the SSCD (assuming it has the magnitude required to produce the large bulk flow) is quite distinct [39] (again because of the long range gravitational potential it induces) and should be detected by Planck. Secondly, the measurement of peculiar velocities via the kinetic Sunyaev-Zel’dovich effect should improve quite significantly with Planck. This should enable testing the claims of [40], which are based on data from WMAP, and determine if the bulk flow is indeed anomalous at even larger scales (which will increase the significance dramatically), and in which direction it points. If our scenario is correct then as one increases the size of the survey, the usual Λ CDM effects should become more negligible compared to the SSCD effect and the measured bulk flow will be more aligned with the giant rings.

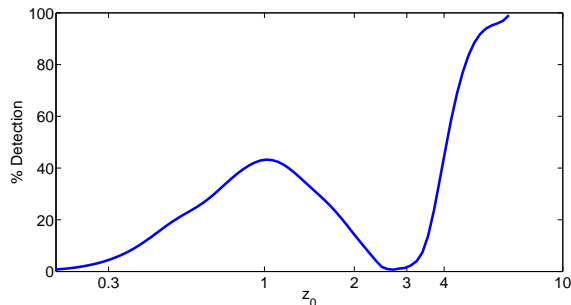


FIG. 7: Percentage of detection (to within a distance of 8°) of a hidden SSCD at different redshifts z_0 by the rings score.

Acknowledgments

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